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Multi-State Selective Maintenance Decisions

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14. ABSTRACT

The primary objective of this research was to develop a modeling-based methodology for managing selective maintenance decisions when multiple (more than two) system states are possible. First, the research literature for selective maintenance and multi-state analysis is presented. Then, a scenario is defined in which systems in various states of maintenance need must perform a number of different missions. For this scenario, a non-linear mathematical program was formulated and three solution procedures for the optimization problem were explored.

15. SUBJECT TERMS

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Executive Summary

All military organizations depend on the reliable performance of repairable systems for the successful completion of missions. Due to limitations in maintenance resources, a maintenance manager must decide how to allocate available resources. This allocation falls within the domain of selective maintenance. Selective maintenance is defined as the process of identifying the subset of maintenance activities to perform from a set of desired maintenance actions. Previously, researchers have developed a class of mathematical models that can be used to identify selective maintenance decisions for the following scenario – A system has just completed a mission and will begin its next mission soon. Maintenance cannot be performed during missions; therefore, the decision-maker must decide which components to maintain prior to the next mission. The selective maintenance models considered to date treat decision-making for binary-state systems, i.e. all components, subsystems, and the system itself are assumed to be either functioning or failed at any point in time.

The primary objective of this project is to develop a modeling-based methodology for managing selective maintenance decisions when multiple (more than two) system states are possible. First, the research literature for selective maintenance and multi-state analysis is presented. Then, we define a scenario in which systems in various states of maintenance need must perform a number of different missions. For this scenario, we formulate a non-linear mathematical program, and we explore three solution procedures for the optimization problem.

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1. Introduction

All military organizations depend on the reliable performance of repairable systems for the successful completion of missions. The use of mathematical modeling for the purpose of modeling repairable systems and designing optimal maintenance policies for these systems has received an extensive amount of attention in the literature. Unfortunately, the vast majority of this work ignores potential limitations on the resources required to perform maintenance actions. This shortcoming has motivated the development of models for selective maintenance, the process of identifying the subset of actions to perform from a set of desirable maintenance actions. Previously, we have developed a class of mathematical models that can be used to identify selective maintenance decisions for the following scenario – A system has just completed a mission and will begin its next mission soon. Maintenance cannot be performed during missions; therefore the decision-maker must decide which components to maintain prior to the next mission. The selective maintenance models formulated to date are based on the assumption of binary (functioning, failed) component, subsystem and system status. As a result, mission reliability is used as the objective function in the resulting mathematical programming models. We wish to improve upon this approach in two ways. First, it may be more realistic to classify component status using more than two discrete levels (if not some continuous measure). This implies multi-state measures of subsystem and system status as well. Second, the performance of a military system typically can be measured using several measures in addition to mission reliability. All these performance measures are functions of the status of the components. The primary objective of this project is to develop multi-state selective maintenance models that incorporate multi-state component status and multiple measures of system performance.

The activities required to achieve the objective of this project are applied to a set of systems utilized by the US Air Force. First, we define the system structure and appropriate status measures for each component in the system. Second, we identify the resources consumed by maintenance actions, the impact on component status of each potential maintenance action, and the quantity of each resource consumed by each maintenance action. Third, we identify the relevant measures of mission performance and develop functions which capture these measures in terms of the component status values. Fourth, we develop a mathematical formulation of the selective maintenance problem. Finally, we develop solution procedures for solving the selective maintenance problem. We define enumerative solution strategies for smaller problems and heuristic strategies for larger problems.

2. Research Literature Review

In this section, a review of the relevant research literature is presented. We begin with a summary of the selective maintenance literature. Then, we conclude with a summary of the literature related to multi-state reliability analysis.

2.1 Selective Maintenance

This project builds upon the body of knowledge in selective maintenance. Selective maintenance falls within the domain of maintenance modeling and optimization. The use of mathematical modeling for the purpose of modeling repairable systems and designing optimal maintenance policies for these systems has received an extensive amount of attention in the literature [5, 6, 7, 8, 10, 12, 13].

The original study in selective maintenance was performed by Rice *et al.* [9]. They define a system that must complete a series of missions where maintenance is performed only during finite breaks between missions. Due to the limited maintenance time, it may not be possible to repair all failed components before the next mission. A nonlinear, discrete selective maintenance optimization model is developed which is designed to maximize system reliability for the next mission. The numbers of components to repair are the decision variables, and the limitation on maintenance time serves as the primary functional constraint. Due to the complexity of the model, total enumeration is the recommended solution procedure. Given that total enumeration is ineffective for large scenarios, a heuristic selective maintenance procedure is developed.

Cassady *et al.* [1, 2] extend the work of Rice *et al.* [9] in several ways. First, more complex systems are analyzed. Specifically, systems are comprised of independent subsystems connected in series with the individual components in each subsystem connected in any fashion. Next, the selective maintenance model is extended to consider the case where both time and cost

are constrained. This leads to the development of three different selective maintenance models. These models include maximizing system reliability subject to both time and cost constraints; minimizing system repair costs subject to a time constraint and a minimum required reliability level; and minimizing total repair time subject to both cost and reliability constraints.

Cassady *et al.* [3] extend the work of Rice *et al.* [9] in two other ways. First, system components are assumed to have Weibull life distributions. This assumption permits systems to experience an increasing failure rate (IFR) and requires monitoring of the age of components. Second, the selective maintenance model is formulated to include three maintenance actions: minimal repair of failed components, replacement of failed components, and preventive maintenance.

Chen *et al.* [4] extend the work of Rice *et al.* [9] and Cassady *et al.* [1] by considering systems in which each component and the system may be in $K + 1$ possible states, 0, 1, ... , K. They use an optimization model to minimize the total cost of maintenance activities subject to a minimum required system reliability.

Schneider and Cassady [10] formulate an optimization model to extend the work of Rice *et al.* [9] by defining a selective maintenance model for a set of systems that must perform a set of missions with system maintenance performed only between sets of missions. Three models are formulated. The first model maximizes the probability that all systems within the set successfully complete the next mission, where as the second model minimizes the variable cost associated with maintenance. A special case of the second model allows the user to maximize the expected value of the number of successful missions in the next set. The third model permits cancellation of a mission based on costs associated with the risk of failure.

2.2 Multi-State Analysis

Recently a great deal of attention has been given to assessing and optimizing multi-state systems. For a comprehensive literature review of multi-state systems, see Lisnianski and Levitin [14]. However, all of this research focuses on systems with coherent states. States exhibit coherency when there is a spectrum of states with perfectly functioning on one end and completely failed at the other and each state in between is a uniform, incremental gradation or degradation towards being perfectly functioning or completely failed. An example would be tire tread wear. The two ends of the tread wear spectrum are brand new and completely bald tires; these states may be denoted 10 and 0 respectively. The states in-between may be represented by the whole numbers $\{1, 2, \dots, 9\}$ where each state corresponds to the percentage of tread remaining: state 9: 90%, state 8: 80%, etc. Coherent states imply that 10 is better than 9, 9 is better than 8, etc., and that the difference in states is uniform whereby the difference between states 8 and 9 is the same as the difference between states 2 and 3.

This research deals with non-coherent states that are descriptive or qualitative in nature. The difference in the states are not necessarily incrementally increasing or decreasing, but denote which missions the system is capable of performing rather than the reliability of the system or some other performance metric. The state of the system is denoted by a vector of the binary status of all of the subsystems of the systems. Thus far no research has been conducted in optimizing systems with multiple, non-coherent states.

3. Problem Statement

3.1 Scenario of Interest

Consider the following scenario. There are q systems, each comprised of m independent subsystems that are idle and available for maintenance. The state of system i is denoted $\underline{a}_i = (a_{i1}, a_{i2}, \ldots, a_{im})$ where a_{ij} denotes the amount of time required to bring subsystem j of system i into a properly operating condition where $a_{ij} \in \{0, 1, \ldots, \varphi_j\}$. Table 1 displays an example $\underline{a}$ vector ($q = 24$, $m = 41$) that will be used as data for the numerical example we will be presenting throughout this paper.

Some maintenance actions require spare parts or other resources that are not readily available. We capture this by assigning a ready time to each subsystem. The ready time of subsystem j in system i, denoted ρ_{ij}, is the time at which these resources are available and maintenance on the subsystem can begin. The ready time of system i is denoted by the vector $\underline{\rho}_i = (\rho_{i1}, \rho_{i2}, \ldots, \rho_{im})$. Table 2 presents an example $\underline{\rho}$ vector for our numerical example.

3.2 Mission Profile

There are n future missions planned where $n \leq q$. Mission k requires some subset of the systems to be operational. Therefore, the state of mission k is denoted $\underline{s}_k = (s_{k1}, s_{k2}, \ldots, s_{km})$ where

$$s_{kj} = \begin{cases} 1 & \text{if mission } k \text{ requires subsystem } j \\ 0 & \text{otherwise} \end{cases}$$

An example $\underline{s}$ vector ($k = 24$) for our numerical example is presented in Table 3.

Table 1. Example $\underline{a}$ Vector

	System																							
Subsystem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Air Frame	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crew Station System	0	0	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	0
Landing Gear System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Flight Control System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turbofan Power Plant (PW Engines)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Aux Power Plant/JFS	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	7	7	0	0	0	0	0	0
Turbofan Power Plant (GE 110)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Environmental Control System	0	0	0	0	0	0	0	0	0	0	0	0	0	10	50	0	0	0	0	0	0	0	0	0
Electrical Power Supply	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Exterior Lighting	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Interior Lighting	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hydraulic/Pneumatic System	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
Fuel System	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0
Oxygen System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fire Detection System	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
Overheat DetectionSystem	0	0	0	0	0	0	0	2	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
Flight Instrumentals	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	1
Malfunction Analysis & Recording Equipment (CFSDR)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
HF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VHF Communications	0	0	0	0	0	0	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
UHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Interphone System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
IFF	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	9	0	0	0	9	0	92	0	0
Improved Data Modem (IDM)/Situation Awareness data link (SADL)	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Radio Navigation	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Global Positioning System (GPS)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fire Control System	0	2	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	7	0	0	0	0	0	0
Airborne Video System	0	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	20	0	0	0	0	0	0	0
Data Transfer Unit	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Radar Altimeter System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	75	0	0	0	0
Targeting Pod (GTP) System	0	29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Navigation Pod (VP) System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	2
Harm Targeting System (HTS)	0	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.0
Weapons Delivery System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gun System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0
Electronic Counter Measures	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Radar Warning Receiver	0	6	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Chaff/Flare Disp System	8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Emergency Equipment	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Smoke Generation System	0	0	0	0	0	0	31	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Explosive Devices and Components	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2. Example $\underline{\rho}$ Vector

	System																							
Subsystem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Air Frame	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0
Crew Station System	2	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Landing Gear System	0	0	0	0	0	0	0	0	0	0	0	4	0	0	4	0	0	0	0	0	0	0	0	0
Flight Control System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0
Turbofan Power Plant (PW Engines)	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96
Aux Power Plant/JFS	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0
Turbofan Power Plant (GE 110)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Environmental Control System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Electrical Power Supply	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96
Exterior Lighting	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Interior Lighting	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0
Hydraulic/Pneumatic System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fuel System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oxygen System	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fire Detection System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Overheat DetectionSystem	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	4	4	0
Flight Instrumentals	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0
Malfunction Analysis & Recording Equipment (CFSDR)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
HF Communications	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0
UHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Interphone System	0	0	0	0	0	0	0	0	0	4	0	0	0	0	4	0	0	0	0	0	0	0	0	0
IFF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Improved Data Modem (IDM)/Situation Awareness data link (SADL)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
Radio Navigation	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Global Positioning System (GPS)	0	0	0	0	0	0	0	0	0	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0
Fire Control System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Airborne Video System	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Data Transfer Unit	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96
Radar Altimeter System	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	96	0
Targeting Pod (GTP) System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
Navigation Pod (VP) System	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Harm Targeting System (HTS)	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
Weapons Delivery System	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0	0	0
Gun System	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2	0	0	0
Electronic Counter Measures	0	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0
Radar Warning Receiver	0	0	0	0	96	96	0	0	0	0	0	0	0	4	0	0	0	0	0	2	0	0	0	0
Chaff/Flare Disp System	0	0	0	0	0	0	0	96	0	4	96	0	0	0	0	0	2	0	0	0	0	0	0	0
Emergency Equipment	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Smoke Generation System	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0	0	0	0	0
Explosive Devices and Components	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3. Example $\underline{s}$ Vector

Subsystem	Mission FSL	ADC	ASC	ASY	ASN	DSP	TNG	TST
Air Frame	1	1	1	1	1	1	1	1
Crew Station System	1	1	1	1	1	1	1	1
Landing Gear System	1	1	1	1	1	1	1	1
Flight Control System	1	1	1	1	1	1	1	1
Turbofan Power Plant (PW Engines)	1	1	1	1	1	1	1	1
Aux Power Plant/JFS	1	1	1	1	1	1	1	1
Turbofan Power Plant (GE 110)	1	1	1	1	1	1	1	1
Environmental Control System	1	1	1	1	1	1	1	1
Electrical Power Supply	1	1	1	1	1	1	1	1
Exterior Lighting	1	1	1	1	1	1	1	1
Interior Lighting	1	1	1	1	1	1	1	1
Hydraulic/Pneumatic System	1	1	1	1	1	1	1	1
Fuel System	1	1	1	1	1	1	1	1
Oxygen System	1	1	1	1	1	1	1	1
Fire Detection System	1	1	1	1	1	1	1	1
Overheat DetectionSystem	1	1	1	1	1	1	1	1
Flight Instrumentals	1	1	1	1	1	1	1	1
Malfunction Analysis & Recording Equipment (CFSDR)	1	0	0	0	0	0	0	0
HF Communications	1	1	0	0	0	0	1	1
VHF Communications	1	1	1	1	1	1	1	1
UHF Communications	1	1	1	1	1	1	1	1
Interphone System	1	1	1	1	1	1	1	1
IFF	1	1	1	1	1	1	1	1
Improved Data Modem (IDM)/Situation Awareness data link (SADL)	1	1	1	1	1	1	1	1
Radio Navigation	1	1	1	1	1	1	1	1
Global Positioning System (GPS)	1	1	1	1	1	1	1	1
Fire Control System	1	1	1	1	1	1	1	1
Airborne Video System	1	0	0	0	0	0	0	0
Data Transfer Unit	1	1	1	1	1	1	1	1
Radar Altimeter System	1	0	1	0	1	0	1	1
Targeting Pod (GTP) System	1	0	1	0	1	0	1	1
Navigation Pod (VP) System	1	0	1	0	1	0	1	1
Harm Targeting System (HTS)	1	0	0	0	0	1	0	0
Weapons Delivery System	1	1	1	1	1	1	1	1
Gun System	1	1	1	1	0	1	1	0
Electronic Counter Measures	1	1	1	1	1	1	0	1
Radar Warning Receiver	1	1	1	1	1	1	1	1
Chaff/Flare Disp System	1	1	1	1	1	1	1	1
Emergency Equipment	1	1	1	1	1	1	1	1
Smoke Generation System	1	0	0	0	0	0	0	0
Explosive Devices and Components	1	1	1	1	1	1	1	1

3.3 Decision Making I

It must be determined which system should be assigned to each mission. The decision variable

$$x_{ik} = \begin{cases} 1 & \text{if system } i \text{ is assigned to mission } k \\ 0 & \text{otherwise} \end{cases}$$

is used to denote the assignments. It will be assumed that every mission is assigned a system and no system is assigned more than one mission. Once the assignments have been made, the total time required for maintenance related to mission k is

$$p_k = \sum_{i=1}^{q} x_{ik} \left(\underline{s}_k \cdot \underline{a}_i \right)$$

This is generally referred to as the processing time in the scheduling literature. It is the sum over all systems of the dot product of the state vector and the condition vector, given that the assignment is made. The condition of a subsystem may be negated if the mission does not require the subsystem. Likewise the ready time for maintenance related to mission k is

$$r_k = \sum_{i=1}^{q} \left(\max_{j \in \{1, 2, \ldots, m\}} s_{kj} r_{ij} \right) x_{ik}$$

For this problem we assume that the waiting time for parts or various delays occurs concurrently. Therefore, the ready time of the mission is just the highest ready time of subsystem required by the mission.

3.4 Decision Making II

Once the assignments of systems to missions are made, maintenance crews must perform the maintenance (γ denotes the number of crews available at one time). We assume that a crew (1) works on one system at a time, (2) works on a system after it is "ready", and (3) works on a system until all maintenance is finished (i.e. no preemptions).

For each mission, a decision must be made as to when maintenance will begin and by which crew the maintenance will be performed. A second decision variable

$$y_{klt} = \begin{cases} 1 & \text{if mx for mission } k \text{ is initiated by crew } l \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

denotes if maintenance for mission k will be initiated by crew l at time t. Time is considered to be a discrete index where $t \in \{0, 1, \ldots, C_{\max} - 1\}$. The time span ($C_{\max}$) is defined as $\max(C_1, \ldots, C_n)$ where C_j is the completion time of job j. This is equivalent to the completion time of the final job in the system [15]. Since, in our scenario, $C_{\max}$ is a function of decision variables, we will place the following upper bound on $C_{\max}$

$$C_{\max} = \sum_{i=1}^{q} \sum_{j=1}^{m} \left(a_{ij} + \rho_{ij}\right)$$

The following assumptions will be made in the second decision making process: (1) every mission gets a crew, (2) maintenance cannot be started before the mission is "ready", and (3) a crew cannot work on two systems at the same time.

3.5 Objective

The overall objective of this problem is to minimize the total weighted completion time (TWCT) of maintenance on all missions. Weighted completion time is also referred to as weighted flow time in the literature. The problem is actually a combination of an assignment problem and a scheduling problem. Systems must be assigned to missions such that the total maintenance time and delay time due to ready times of the assignment corresponds simultaneously with minimizing the TWCT of the scheduled maintenance.

The scheduling problem is denoted $Pm \mid r_j \mid \sum w_j C_j$ using the notation of Lawler *et al.* [16]. The scheduling problem is considered to have identical parallel machines (*Pm*) with ready

times (r_j) and a TWCT objective. The crews performing maintenance are considered to be the identical parallel machines as it will be assumed that all maintenance crews contain the same number of people and can perform maintenance at the same pace. We are assigning the various missions different weights (w_j) or importance because it may be desired for more important missions to have an earlier completion time (C_j). A larger weight implies more importance. Weights for our numerical example are presented in Table 4. The $Pm \mid r_j \mid \sum w_j C_j$ is strongly NP-hard.

Table 4. Example Weights (w_k)

Mission Type	w_k
FSL	8
ADC	6
ASC	10
ASY	5
ASN	9
DSP	3
TNG	1
TST	2

4. Motivation

This research is motivated by a real-world Air Force scenario in which planes can have multiple qualitative states. The system under consideration is the F-16 A/B/C/D due to our experience with them at Hill AFB, UT. There are 41 subsystems for the F-16 as detailed by the mission essential subsystem list (MESL) (AFI121-103_ACCSUP1). There are also eight possible missions for an F-16:

1. FSL – Full System List
2. ADC – Air Defense, Conventional
3. ASC - Air to Surface, Conventional
4. ASY – Air Superiority
5. ASN - Air to Surface, Nuclear
6. DSP – Defense Suppression
7. TNG - Training
8. TST – Testing

Mission FSL denotes that all subsystems are required. The remaining seven missions require some combination of the 41 subsystems, but not all. The qualitative state of each plane is which missions it is capable of performing.

5. Mathematical Formulation

Sets and Indexes

$i \equiv$ system number ($i = 1, 2, \ldots, q$)
$j \equiv$ subsystem number ($j = 1, 2, \ldots, m$)
$k \equiv$ mission number ($k = 1, 2, \ldots, n$)
$l \equiv$ crew number ($l = 1, 2, \ldots, \gamma$)
$t \equiv$ time ($t = 0, 1, \ldots, C_{\max} - 1$)

Parameters

$a_{ij} \equiv$ amount of time required to bring subsystem j of system i into a properly operating condition

$$s_{kj} \equiv \begin{cases} 1 & \text{if mission } k \text{ requires subsytem } j \\ 0 & \text{otherwise} \end{cases}$$

$\rho_{ij} \equiv$ ready time of subsystem j in system i
$w_k \equiv$ importance (weight) of mission k

Decision Variables

$$x_{ik} \equiv \begin{cases} 1 & \text{if system } i \text{ is assigned to mission } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{klt} \equiv \begin{cases} 1 & \text{if maintenance for mission } k \text{ is initiated by crew } l \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

5.1 Mathematical Program

minimize $$\sum_{k=1}^{n} \mathrm{w}_k \left\{ \sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} \left[\mathrm{t} + \sum_{i=1}^{q} \left(x_{ik} \sum_{j=1}^{m} a_{ij} s_{kj} \right) \right] \right\} y_{klt}$$

subject to:

$$\sum_{i=1}^{q} x_{ik} = 1 \qquad k = 1,2,\ldots,n \qquad (1)$$

$$\sum_{k=1}^{n} x_{ik} \le 1 \qquad i = 1,2,\ldots,q \qquad (2)$$

$$\sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} y_{klt} = 1 \qquad k = 1,2,\ldots,n \qquad (3)$$

$$\sum_{k=1}^{n} \sum_{t'=\max(t-p_k,0)}^{t-1} y_{klt'} \le 1 \qquad l = 1,2,\ldots,\gamma;\ t = 0,1,\ldots,C_{\max}-1 \qquad (4)$$

$$c_k - p_k \ge r_k \qquad k = 1,2,\ldots,n \qquad (5)$$

$$r_k = \sum_{i=1}^{q} \left(\max_{j \in \{1,2,\ldots,m\}} s_{kj} \rho_{ij} \right) x_{ik} \qquad k = 1,2,\ldots,n \qquad (6)$$

$$c_k = \sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} \left[\mathrm{t} + \sum_{i=1}^{q} \left(x_{ik} \sum_{j=1}^{m} a_{ij} s_{kj} \right) \right] y_{klt} \qquad k = 1,2,\ldots,n \qquad (7)$$

$$p_k = \sum_{i=1}^{q} \left(x_{ik} \sum_{j=1}^{m} a_{ij} s_{kj} \right) \qquad k = 1,2,\ldots,n \qquad (8)$$

$$C_{\max} = \sum_{i=1}^{q} \sum_{j=1}^{m} \left(a_{ij} + \rho_{ij} \right) \qquad (9)$$

$$x_{ik} = 0 \text{ or } 1 \qquad i = 1,2,\ldots,q;\ k = 1,2,\ldots,n \qquad (10)$$

$$y_{klt} = 0 \text{ or } 1 \qquad k = 1,2,\ldots,n;\ l = 1,2,\ldots,\gamma;\ t = 0,1,\ldots,C_{\max}-1 \qquad (11)$$

The above formulation represents the non-linear binary program for the multi-state selective maintenance problem. The formulation is constructed using time-indexed variables. The objective function minimizes TWCT where the completion time of mission k is given by (7) in the list of constraints. Constraint (1) says that every mission is assigned a system, and likewise (2) ensures that no system gets more than one mission. Constraint (3) ensures that every mission

is assigned a crew. Constraint (4) does not allow preemption or the ability for a crew to work on two systems at the same time. Constraint (5) enforces the ready times of the missions by ensuring that the completion time of the mission minus its processing time is greater than or equal to the ready time of the mission. Constraints (6-7) set the ready time and completion time of the missions, respectively as previously defined in the problem statement. Constraints (10-11) enforce the binary restriction of the decision variables.

As previously, mentioned the scheduling problem alone is strongly NP-hard and therefore the entire problem is strongly NP-hard. With the combination assignment problem and scheduling problem the formulation is non-linear both in the objective function and in the constraints. Non-linearity occurs in the objective function when the two decision variables are multiplied together when the summations are expanded. This specific case of non-linearity is known as a quadratic objective function. Non-linearity is also introduced in constraint (4) when the lower bound of the inner summation contains the variable p_k which is a function of the decision variable x_{ik}.

6. Solution Procedure

Three solution approaches for the multi-state selective maintenance problem were investigated. First, the problem was solved using a total enumeration procedure. Second, a combination of a heuristic and commercial solver was utilized. Finally, a dispatching rule was tested.

6.1 Total Enumeration

The total enumeration procedure was coded using Visual Basic for Applications (VBA) running behind a Microsoft Excel interface. It is an exponential time procedure that first enumerates all of the possible assignments and for each assignment enumerates all possible schedules and reports the minimum objective value found.

Due to the computational complexity of the multi-state selective maintenance problem, only extremely small problem instances can be solved in a reasonable amount of time using total enumeration. The number of iterations required to enumerate the multi-state selective maintenance problem is $10^{2n\gamma}$. Thus, for a three mission, three, system, two crews scenario, one-trillion iterations are required. A Pentium IV 2.0 GHz with 2.0 GB of RAM took weeks to enumerate this small problem instance. Obviously, a scenario this small is not useful for application by the Air Force and even if it was, this is not a practical computation time for implementation. Therefore, other solutions approaches must be explored.

6.2 Heuristic and Commercial Solver

A commercial solver has been utilized to facilitate the solution process. Unfortunately, the formulation is a non-linear program which makes it incapable of being solved by most almost all commercial solver packages. Some packages are capable of solving quadratic programs as our problem formulation contains a quadratic objective function. However, the non-linear constraint

(4) eliminates the quadratic properties of the problem. Therefore, we have implemented a heuristic in combination with a commercial solver to obtain solutions to the multi-state selective maintenance problem. The heuristic simply breaks the multi-state selective maintenance problem up into two separate subproblems: an assignment problem and a scheduling problem. The assignment problem will be solved to optimality using total enumeration for a ready-time-based weighted shortest processing time (WSPT) first objective function. The heuristic is based on the famous rule in scheduling theory that jobs that are ordered in decreasing order of w_j / p_j is optimal for the single machine TWCT ($1 \| \sum w_j C_j$) problem. Our problem deviates from this simplified problem in that it is a more complex version containing ready-times and multiple parallel machines. However, the modified objective function will still serve to reduce the TWCT objective. The heuristic formulates assignments of systems to missions by making the assignment with the highest

$$\frac{w_k}{p_k + \sqrt{r_k}} = \frac{w_k}{\sum_{i=1}^{q} x_{ik}\left(\underline{s}_k \cdot \underline{a}_i\right) + \sqrt{\sum_{i=1}^{q}\left(\max_{j \in \{1,2,\ldots,m\}} s_{kj} r_{ij}\right) x_{ik}}}$$

And continuing in this fashion until all missions have been assigned a system and not more than one system is assigned a mission. The heuristic is denoted a read-time based WSPT because of the addition of r_k in the denominator. It is desired to have the missions with the highest w_k / p_k be scheduled as soon as possible so the ready time is placed in the denominator to have it minimized. The square root of the ready-time is taken so it does not dominate the processing time. Ready-times will often be several orders of magnitudes higher than the processing time as we are measuring time in hours and ready times may be several days. The heuristic for the assignment problem was coded in VBA and runs in a fraction of a second. Table 6 demonstrates the assignment made by the heuristic for our numerical example data we have presented

previously. For our example, consider the distribution of the number of each missions type presented in Table 5. The assignment column in Table 6 shows which system is assigned to which mission type. The job formulation column displays the resulting job parameters of the assignment. After the assignment has been made, each job will have an associated weight (w_j), ready time (r_j), and processing time (p_j)

Table 5. Numerical Example Mission Type Distribution

Mission Type	**#**
FSL	7
ADC	3
ASC	4
ASY	5
ASN	1
DSP	2
TNG	2
TST	0

Table 6. Numerical Example Heuristic Assignment

Assignment		Job Formulation			
System	**Mission**	**#**	w_j	r_j	p_j
13	FSL	1	1	96	0
21	ADC	2	1	96	0
19	FSL	3	10	0	1
10	FSL	4	9	4	1
9	FSL	5	10	0	4
23	FSL	6	6	4	2
17	ASY	7	10	2	7
12	FSL	8	10	4	8
1	ASC	9	8	2	9
3	ASN	10	8	4	11
4	FSL	11	8	96	1
5	ASC	12	8	96	3
11	TNG	13	8	96	4
8	ASY	14	8	96	2
6	ASY	15	6	96	4
2	ADC	16	6	0	8
16	ASC	17	3	0	11
24	ASY	18	3	96	1
22	ASC	19	5	96	92
20	ASY	20	5	2	21
18	TNG	21	5	96	15
15	DSP	22	5	4	53
14	ADC	23	5	96	10
7	DSP	24	5	96	51

The heuristic solution to the assignment problem will then be used as an input to the scheduling problem which is formulated in AMPL and solved by ILOG's CPLEX 9.0. CPLEX can obtain the optimal solution to the scheduling problem, however, a global optimal solution for the aggregated problem cannot be guaranteed, because the solution of the assignment problem is not necessarily the assignment for the global optimum of the entire problem. The mathematical formulation of the scheduling problem is a subset of the entire multi-state selective maintenance formulation and is presented below. The AMPL formulation is contained in Appendix A.

Sets and Indexes
$k \equiv$ mission number (k = 1, 2, . . . , n)
$l \equiv$ machine number (l = 1, 2, . . . , γ)
$t \equiv$ time (t = 0, 1, . . . , $C_{\max}$ -1)

Parameters
$p_k \equiv$ processing time of job k
$w_k \equiv$ weight of job k

Decision Variable

$$x_{ikt} \equiv \begin{cases} 1 & \text{if job } k \text{ starts processing at time t by crew l} \\ 0 & \text{otherwise} \end{cases}$$

minimize $$\sum_{k=1}^{n} w_k \sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} (t + p_j) x_{klt}$$

subject to:

$$\sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} x_{klt} = 1 \qquad k = 1,2,\ldots,n$$

$$\sum_{k=1}^{n} \sum_{t'=\max(t-p_{j,0})}^{t-1} x_{klt'} \leq 1 \qquad \begin{array}{l} 1 = 1,2,\ldots,\gamma \\ t = 0,1,\ldots,C_{\max} - 1 \end{array}$$

$$\sum_{l=1}^{\gamma} \sum_{t=0}^{C_{\max}-1} t \cdot x_{klt} = s_k \qquad k = 1,2,\ldots,n$$

$$s_k \geq r_k \qquad k = 1,2,\ldots,n$$

$$x_{ijt} = 0 \text{ or } 1 \qquad \begin{array}{l} k = 1,2,\ldots,n \\ t = 0,1,\ldots,C_{\max} - 1 \end{array}$$

The optimal crew assignments and schedule obtained by CPLEX for our numerical example is displayed in Table 7. The TWCT objective function for our numerical example given by the heuristic/CPLEX solution method is 7,992.

Table 7. Numerical Example: CPLEX Crew Assignments and Schedule

Crew #	Job #	Start Time (t)
1	9	2
	12	96
	19	99
2	7	2
	20	9
	15	96
3	16	0
	22	8
	14	96
	21	98
4	3	0
	4	4
	8	5
	18	96
	24	97
5	5	0
	6	4
	10	6
	11	96
	23	97
6	17	0
	13	96
	1	n/a
	2	n/a

6.3 Dispatching Rule

The dispatching rule is a simplified version of the heuristic presented above without the CPLEX optimization. The dispatching rule is designed to be easily calculated by hand and implemented in the field without the aid of a computer. Missions are simply "dispatched" or scheduled in order of the decreasing objective function

$$\frac{w_k}{\sum_{j=1}^{m} a_{ij} + \sqrt{\sum_{j=1}^{m} r_{ij}}}$$

For each system i the total processing times of all m subsystems are summed and added to the square root of the sum of all of the ready times of each subsystem j on system i. Missions are dispatched by taking the mission with the highest weight, matching it with the system with the lowest

$$\sum_{j=1}^{m} a_{ij} + \sqrt{\sum_{j=1}^{m} r_{ij}}$$

and then scheduling it with the crew with the least number of jobs currently in queue. This is continued until all missions have been scheduled. The dispatching rule does not account for what subsystems are required by the mission in making the system assignments. It just encapsulates all processing times and ready times into a sum regardless of whether they will be required by the mission in the assignment. Table 8 presents the dispatching rule's assignment and schedule for our numerical example. The TWCT objective function for the dispatching rule on our numerical example is 11,309 which is 41.5% higher than the objective given by heuristic solution approach.

Table 8. Dispatching Rule for Numerical Example

Mission	Weight (w_j)	$\sum_{j=1}^{m} a_{ij} + \sqrt{\sum_{j=1}^{m} r_{ij}}$	System	Crew
ASC	10	2.41	19	1
ASC	10	4	9	2
ASC	10	6.74	10	3
ASC	10	9.90	21	4
ASN	9	11	1	5
FSL	8	11	16	6
FSL	8	12.10	8	1
FSL	8	12.80	5	2
FSL	8	13	3	3
FSL	8	13.93	13	4
FSL	8	14.86	4	5
FSL	8	18	11	6
ADC	6	18	12	1
ADC	6	19.30	23	2
ADC	6	19.97	24	3
ASY	5	20	14	4
ASY	5	20.97	6	5
ASY	5	28.41	17	6
ASY	5	28.86	18	1
ASY	5	56.16	15	2
DSP	3	58	2	3
DSP	3	95.86	7	4
TNG	1	97.41	20	5
TNG	1	102.20	22	6

7. Experimental Design

Realistic problem instances of the multi-state selective maintenance problem were randomly generated and both the heuristic/optimization and dispatching rule approaches were tested for their performance in terms of solution quality (as measured by the TWCT objective) and computation time.

7.1 Number of Systems, Missions, and Crews

We will be using the F-16 as our motivating example as outlined above in section 4. Numerical examples will be evaluated at the squadron level (24 planes). All scenarios will be assumed to have 6 identical crews available for maintenance at any given time. In each scenario the number of systems is considered to be equal to the number of missions although in actuality the number of systems may exceed the number of missions, but never fall below. Thus, there will be 24 missions; each, one of the eight different missions types outlined above. The number of each mission type will be distributed according to Table 9.

Table 9. Mission Type Distribution

Mission Type	# of Missions
FSL	~DU[1, 24]
ADC	~DU[1, 24 - #FSL]
ASC	~DU[1, 24 – (#FSL + #ADC)]
ASY	~DU[1, 24 – (#FSL + #ADC + #ASC)]
ASN	~DU[1, 24 – (#FSL + #ADC + #ASC + #ASY)]
DSP	~DU[1, 24 – (#FSL + #ADC + #ASC + #ASY + #ASN)]
TNG	~DU[1, 24 – (#FSL + #ADC + #ASC + #ASY + #ASN + #DSP)]
TST	24 – (#FSL + #ADC + #ASC + #ASY + #ASN + #DSP + #TNG)

By using the discrete uniform (DU) distribution we ensure that the number of missions takes on whole number values. By ordering the distribution in this way we are, in effect, weighting the first missions higher because there is a higher likelihood that they will have a greater number of missions. Whereas the latter missions (e.g. TNG and TST) will likely have a very small number

of missions or none at all because all of the 24 missions will already be allocated by the time these missions receive their assignments. We believe this ordering to be consistent with the types of missions required by the Air Force, with the majority of missions requiring the Full System List (FSL) and tapering down to only a few Training (TNG) and Testing (TST) missions.

7.2 State of the System

For each subsystem for a given system, whether the subsystem was functioning or failed upon returning from a mission is considered to be a Bernoulli trial with probability of functioning $p = 0.95$. Therefore, each subsystem for a given system has a 5% chance of being failed. Assuming a series system for an aircraft with 41 subsystems, this would mean that the F-16 only has a $0.95^{41} = 0.1221$ total reliability which is much lower than actuality. A true F-16 has an estimated reliability of approximately 0.90. This would mean that each individual subsystem would actually have a $0.90^{1/41} = 0.9974$ reliability given a series system. However, for the purpose of our experiment, a 0.9974 subsystem reliability is too high and does not generate enough failures to test our solution approaches, therefore 0.95 was used.

If a subsystem was determined to be failed by the Bernoulli trial, the number of hours required to bring the subsystem to a fully functioning state was determined by a lognormal distribution. It was desired to have a distribution whose mean was 4 hours but still had a 5% chance of maintenance times that exceeded 24 hours. This was designed to capture the fact that the average repair time for an F-16 requires four hours. Some repairs may only take an hour as a failed component may be easily isolated and replaced. However, there is a chance that an engine failure may occur or a problem may take a very long time to troubleshoot, therefore we allow for the small possibility of a very long maintenance time. The lognormal distribution was chosen for it skewed shaped and that it only yields positive values. Lognormal distribution parameters with

an exact mean of 4 and a $\Pr(X \leq 24) = 0.05$ could not be obtained because imaginary roots were obtained when solving the systems of equations. The closest real distribution values that can be obtained are $\mu = 0.29034$ $\sigma = 1.7608$ which gives a mean of approximately 6.3. Thus, the time to bring a failed subsystem to a functioning state is distributed LN(0.29034, 3.1004). Because the lognormal distribution is continuous, and we desire to have integer values, the ceiling of all values of the random distribution is taken.

7.3 Ready Times

Ready times are assumed to follow a custom made discrete distribution as follows

$$r_{ij} \sim \begin{cases} 0\text{ hours} & p = 0.1 \\ 2\text{ hours} & p = 0.3 \\ 4\text{ hours} & p = 0.3 \\ 96\text{ hours} & p = 0.3 \end{cases}$$

It is assumed that if a spare parts delay is incurred it will fall under one of the four scenarios outlined above. There is a 10% chance that a spare part will not be needed or it is immediately available on hand to maintenance personnel. The 2-hour spare part delay scenario is designed to model a part stored in a warehouse on the base that must be located and transported to the maintenance hangar. The 2-hour scenario is assumed to occur 30% of the time. The 4-hour scenario models the cannibalization of a part from another plane. It is assumed to take approximately 4 hours for the part to be extracted from the cannibalized aircraft and installed in the aircraft being maintained. This scenario is also considered to occur 30% of the time. The final scenario, also with a 30% probability, is if the spare part need is not available on-site and must be shipped from a depot in a foreign location. It is believed to take approximately 96 hours (4 days) to receive a part from the depot, and thus delay maintenance actions for that amount of time.

7.4 Weights

The weight of each mission was distributed DU[1, 10]. Thus each mission was assigned a whole number weight value between 1 and 10, inclusively. Different missions were allowed to have the same weight.

8. Results

There were 233 replications of the above experimental design generated and solved using both the heuristic/optimization and the dispatching rule solution approaches. The heuristic coded in VBA ran in a fraction of a second, however the optimization of the scheduling problem using CPLEX took an average of 7.5 minutes with the longest solution times taking over 70 minutes. The dispatching rule runs in a fraction of a second and is able to obtain solutions that are on average a mere 0.33% above the heuristic/optimization approach and even beat the heuristic/optimization approach in 86 out of the 233 experiments.

It is impossible to determine how the heuristic/optimization and dispatching rule approaches compare to the optimal solution because total enumeration is infeasible for practical size problem instances. It is reasonable to conclude that both solutions approaches are providing quality solutions due to the similarity of the results.

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Appendix A

File name: MSSMx_Solution_Code.xls

1. *Setting up the spreadsheet for use*

When you open the file, you may receive a message similar to the one shown in Figure A.1.

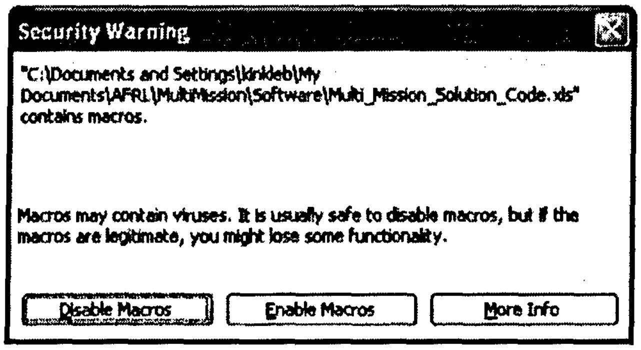
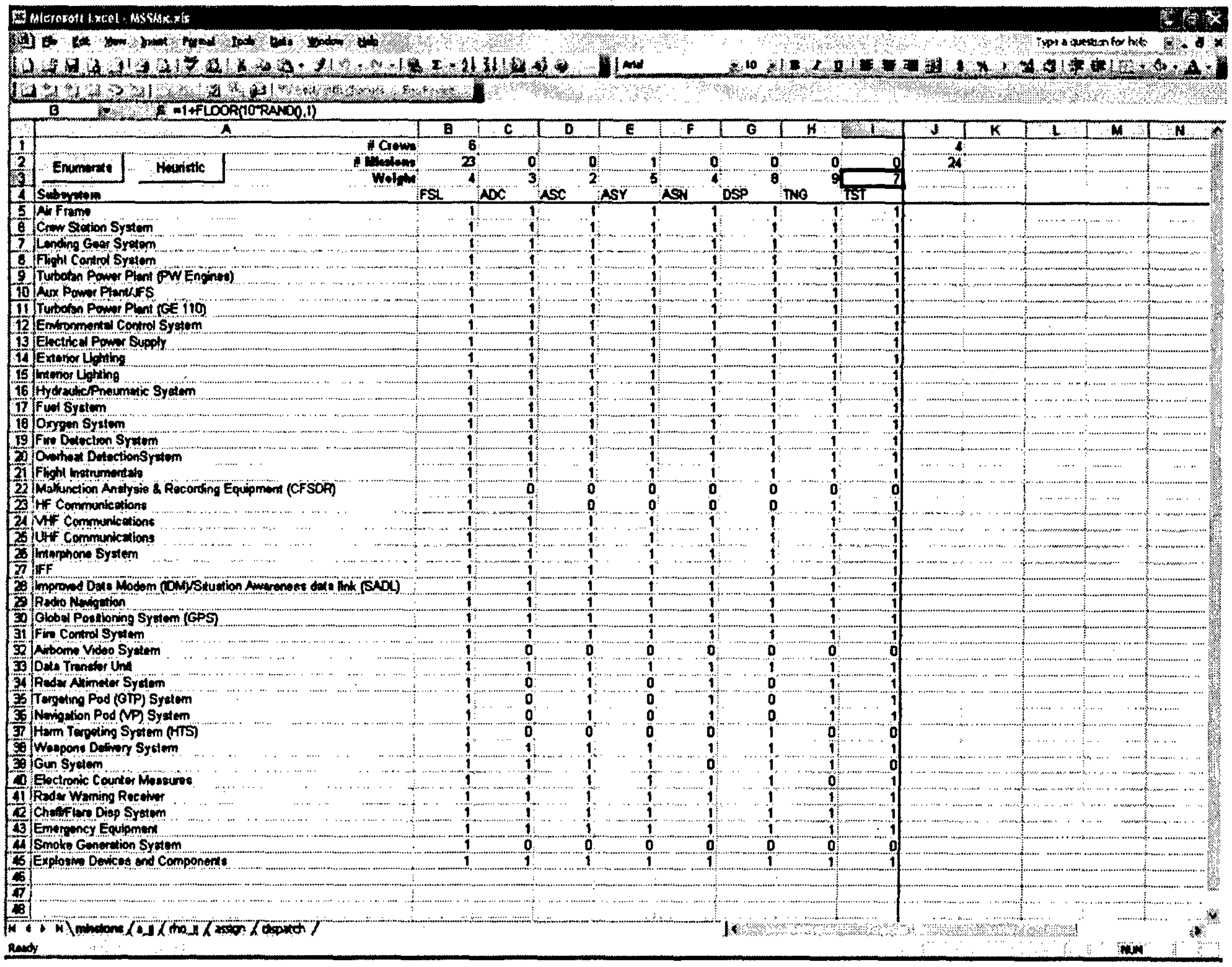


Figure A.1 Macro Notification

The Visual Basic code used to evaluate the model is written within macros. Therefore, you should click on "Enable Macros." This will open the "missions" worksheet shown in Figure A.2.

2. *Inputting Data*

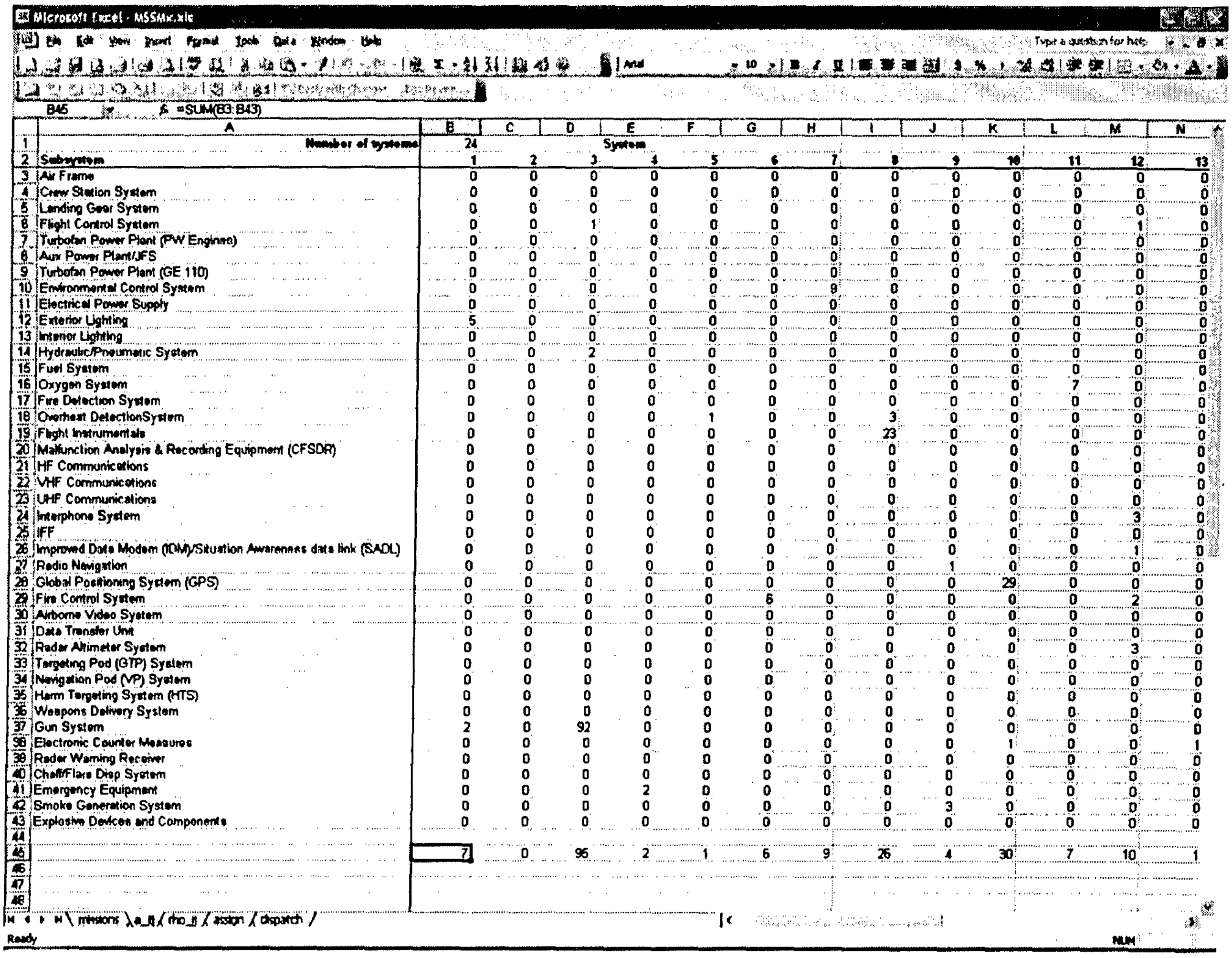
This "missions" worksheet (Figure A.2) displays the subsystems required by each mission. Next, enter the number of crews available to perform maintenance at any given time. Then enter the number of missions of each type needed to be performed in the row of the corresponding three-letter mission abbreviation in the field "# Missions". Likewise, enter the weight or importance of the mission in the "Weights" field. The weight should be a whole number.



	A	B	C	D	E	F	G	H	I	J
1	# Crews	6								4
2	Enumerate Heuristic # Missions	23	0	0	1	0	0	0	0	24
3	Weight	4	3	2	5	4	8	9	7	
4	Subsystem	FSL	ADC	ASC	ASY	ASN	DSP	TNG	TST	
5	Air Frame	1	1	1	1	1	1	1	1	
6	Crew Station System	1	1	1	1	1	1	1	1	
7	Landing Gear System	1	1	1	1	1	1	1	1	
8	Flight Control System	1	1	1	1	1	1	1	1	
9	Turbofan Power Plant (PW Engines)	1	1	1	1	1	1	1	1	
10	Aux Power Plant/JFS	1	1	1	1	1	1	1	1	
11	Turbofan Power Plant (GE 110)	1	1	1	1	1	1	1	1	
12	Environmental Control System	1	1	1	1	1	1	1	1	
13	Electrical Power Supply	1	1	1	1	1	1	1	1	
14	Exterior Lighting	1	1	1	1	1	1	1	1	
15	Interior Lighting	1	1	1	1	1	1	1	1	
16	Hydraulic/Pneumatic System	1	1	1	1	1	1	1	1	
17	Fuel System	1	1	1	1	1	1	1	1	
18	Oxygen System	1	1	1	1	1	1	1	1	
19	Fire Detection System	1	1	1	1	1	1	1	1	
20	Overheat DetectionSystem	1	1	1	1	1	1	1	1	
21	Flight Instrumentals	1	1	1	1	1	1	1	1	
22	Malfunction Analysis & Recording Equipment (CFSDR)	1	0	0	0	0	0	0	0	
23	HF Communications	1	1	0	0	0	0	1	1	
24	VHF Communications	1	1	1	1	1	1	1	1	
25	UHF Communications	1	1	1	1	1	1	1	1	
26	Interphone System	1	1	1	1	1	1	1	1	
27	IFF	1	1	1	1	1	1	1	1	
28	Improved Data Modem (IDM)/Situation Awareness data link (SADL)	1	1	1	1	1	1	1	1	
29	Radio Navigation	1	1	1	1	1	1	1	1	
30	Global Positioning System (GPS)	1	1	1	1	1	1	1	1	
31	Fire Control System	1	1	1	1	1	1	1	1	
32	Airborne Video System	1	0	0	0	0	0	0	0	
33	Data Transfer Unit	1	1	1	1	1	1	1	1	
34	Radar Altimeter System	1	0	1	0	1	0	1	1	
35	Targeting Pod (GTP) System	1	0	1	0	1	0	1	1	
36	Navigation Pod (VP) System	1	0	1	0	1	0	1	1	
37	Harm Targeting System (HTS)	1	0	0	0	0	1	0	0	
38	Weapons Delivery System	1	1	1	1	1	1	1	1	
39	Gun System	1	1	1	1	0	1	1	0	
40	Electronic Counter Measures	1	1	1	1	1	1	0	1	
41	Radar Warning Receiver	1	1	1	1	1	1	1	1	
42	Chaff/Flare Disp System	1	1	1	1	1	1	1	1	
43	Emergency Equipment	1	1	1	1	1	1	1	1	
44	Smoke Generation System	1	0	0	0	0	0	0	0	
45	Explosive Devices and Components	1	1	1	1	1	1	1	1	

Figure A.2 Missions Worksheet

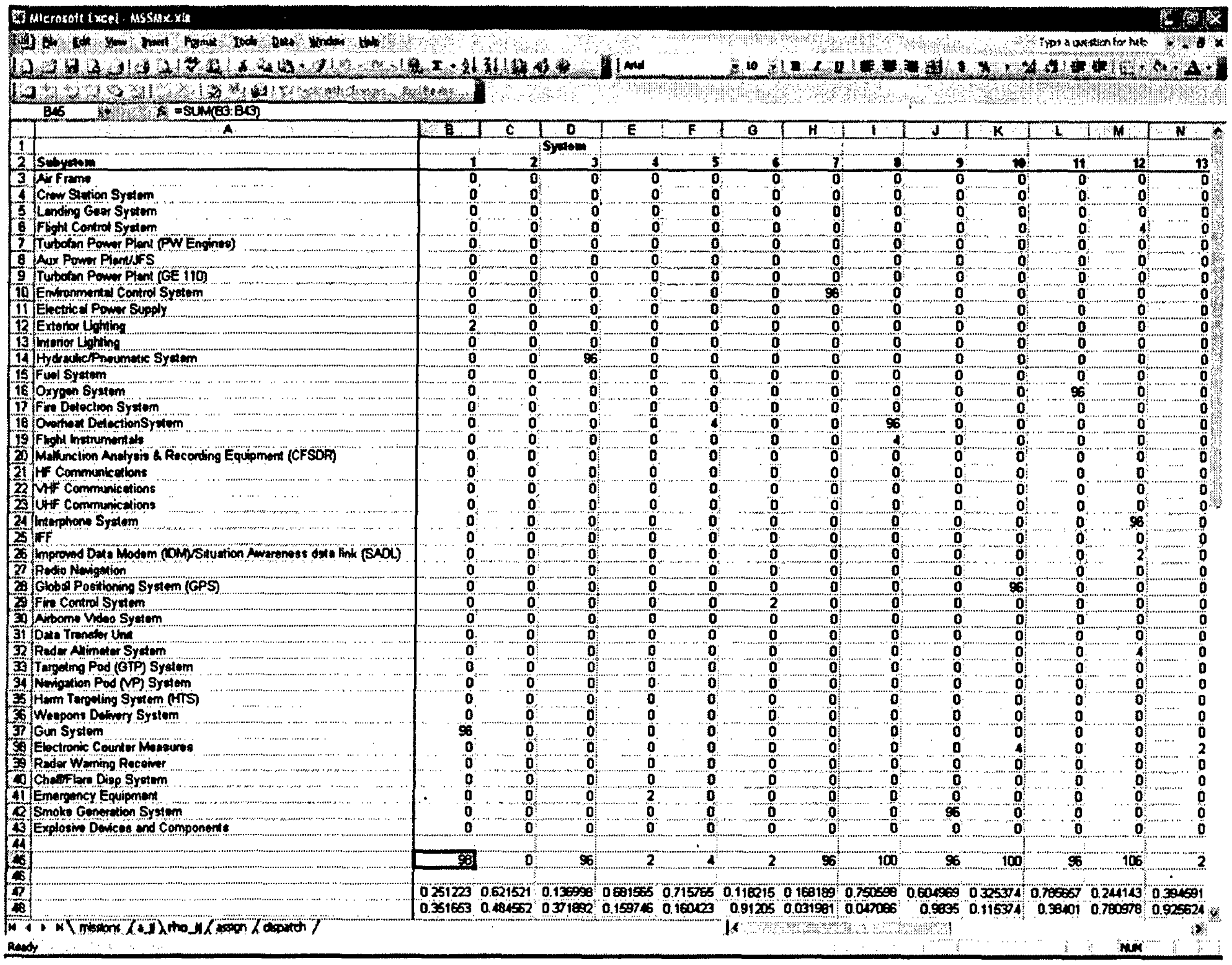
To enter number of systems available and the status of each system, click on the a_ij worksheet tab to display the a_ij worksheet shown in Figure A.3. Enter the number of systems available in the "Number of Systems Field." Next, enter the number of maintenance hours required to bring the subsystem of a given system to a fully functioning state. This should be a whole number value. Match the column of the corresponding system number with the row of the subsystem requiring maintenance.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Number of systems	24			System									
2	Subsystem	1	2	3	4	5	6	7	8	9	10	11	12	13
3	Air Frame	0	0	0	0	0	0	0	0	0	0	0	0	0
4	Crew Station System	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Landing Gear System	0	0	0	0	0	0	0	0	0	0	0	0	0
6	Flight Control System	0	0	1	0	0	0	0	0	0	0	0	1	0
7	Turbofan Power Plant (PW Engines)	0	0	0	0	0	0	0	0	0	0	0	0	0
8	Aux Power Plant/JFS	0	0	0	0	0	0	0	0	0	0	0	0	0
9	Turbofan Power Plant (GE 110)	0	0	0	0	0	0	0	0	0	0	0	0	0
10	Environmental Control System	0	0	0	0	0	0	9	0	0	0	0	0	0
11	Electrical Power Supply	0	0	0	0	0	0	0	0	0	0	0	0	0
12	Exterior Lighting	5	0	0	0	0	0	0	0	0	0	0	0	0
13	Interior Lighting	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Hydraulic/Pneumatic System	0	0	2	0	0	0	0	0	0	0	0	0	0
15	Fuel System	0	0	0	0	0	0	0	0	0	0	0	0	0
16	Oxygen System	0	0	0	0	0	0	0	0	0	0	7	0	0
17	Fire Detection System	0	0	0	0	0	0	0	0	0	0	0	0	0
18	Overheat DetectionSystem	0	0	0	0	1	0	0	3	0	0	0	0	0
19	Flight Instrumentals	0	0	0	0	0	0	0	23	0	0	0	0	0
20	Malfunction Analysis & Recording Equipment (CFSDR)	0	0	0	0	0	0	0	0	0	0	0	0	0
21	HF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
22	VHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
23	UHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
24	Interphone System	0	0	0	0	0	0	0	0	0	0	0	3	0
25	IFF	0	0	0	0	0	0	0	0	0	0	0	0	0
26	Improved Data Modem (IDM)/Situation Awareness data link (SADL)	0	0	0	0	0	0	0	0	0	0	0	1	0
27	Radio Navigation	0	0	0	0	0	0	0	0	1	0	0	0	0
28	Global Positioning System (GPS)	0	0	0	0	0	0	0	0	0	29	0	0	0
29	Fire Control System	0	0	0	0	0	6	0	0	0	0	0	2	0
30	Airborne Video System	0	0	0	0	0	0	0	0	0	0	0	0	0
31	Data Transfer Unit	0	0	0	0	0	0	0	0	0	0	0	0	0
32	Radar Altimeter System	0	0	0	0	0	0	0	0	0	0	0	3	0
33	Targeting Pod (GTP) System	0	0	0	0	0	0	0	0	0	0	0	0	0
34	Navigation Pod (VP) System	0	0	0	0	0	0	0	0	0	0	0	0	0
35	Harm Targeting System (HTS)	0	0	0	0	0	0	0	0	0	0	0	0	0
36	Weapons Delivery System	0	0	0	0	0	0	0	0	0	0	0	0	0
37	Gun System	2	0	92	0	0	0	0	0	0	0	0	0	0
38	Electronic Counter Measures	0	0	0	0	0	0	0	0	0	1	0	0	1
39	Radar Warning Receiver	0	0	0	0	0	0	0	0	0	0	0	0	0
40	Chaff/Flare Disp System	0	0	0	0	0	0	0	0	0	0	0	0	0
41	Emergency Equipment	0	0	0	2	0	0	0	0	0	0	0	0	0
42	Smoke Generation System	0	0	0	0	0	0	0	0	3	0	0	0	0
43	Explosive Devices and Components	0	0	0	0	0	0	0	0	0	0	0	0	0
44														
45		7	0	96	2	1	6	9	26	4	30	7	10	1
46														
47														
48														

Figure A.3 a_ij Worksheet

Next, click on the rho_ij tab at the bottom of the screen to display the rho_ij worksheet show in Figure A.4. This worksheet will allow the user to input the ready times of the subsystems for each mission. Ready times should be whole number values and entered exactly as the maintenance hours described above for worksheet a_ij.



Microsoft Excel - MSSMx.xls

B45 =SUM(B3:B43)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1				System										
2	Subsystem	1	2	3	4	5	6	7	8	9	10	11	12	13
3	Air Frame	0	0	0	0	0	0	0	0	0	0	0	0	0
4	Crew Station System	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Landing Gear System	0	0	0	0	0	0	0	0	0	0	0	0	0
6	Flight Control System	0	0	0	0	0	0	0	0	0	0	0	4	0
7	Turbofan Power Plant (PW Engines)	0	0	0	0	0	0	0	0	0	0	0	0	0
8	Aux Power Plant/JFS	0	0	0	0	0	0	0	0	0	0	0	0	0
9	Turbofan Power Plant (GE 110)	0	0	0	0	0	0	0	0	0	0	0	0	0
10	Environmental Control System	0	0	0	0	0	0	96	0	0	0	0	0	0
11	Electrical Power Supply	0	0	0	0	0	0	0	0	0	0	0	0	0
12	Exterior Lighting	2	0	0	0	0	0	0	0	0	0	0	0	0
13	Interior Lighting	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Hydraulic/Pneumatic System	0	0	96	0	0	0	0	0	0	0	0	0	0
15	Fuel System	0	0	0	0	0	0	0	0	0	0	0	0	0
16	Oxygen System	0	0	0	0	0	0	0	0	0	0	96	0	0
17	Fire Detection System	0	0	0	0	0	0	0	0	0	0	0	0	0
18	Overheat DetectionSystem	0	0	0	0	4	0	0	96	0	0	0	0	0
19	Flight Instrumentals	0	0	0	0	0	0	0	4	0	0	0	0	0
20	Malfunction Analysis & Recording Equipment (CFSDR)	0	0	0	0	0	0	0	0	0	0	0	0	0
21	HF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
22	VHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
23	UHF Communications	0	0	0	0	0	0	0	0	0	0	0	0	0
24	Interphone System	0	0	0	0	0	0	0	0	0	0	0	96	0
25	IFF	0	0	0	0	0	0	0	0	0	0	0	0	0
26	Improved Data Modem (IDM)/Situation Awareness data link (SADL)	0	0	0	0	0	0	0	0	0	0	0	2	0
27	Radio Navigation	0	0	0	0	0	0	0	0	0	0	0	0	0
28	Global Positioning System (GPS)	0	0	0	0	0	0	0	0	0	96	0	0	0
29	Fire Control System	0	0	0	0	0	2	0	0	0	0	0	0	0
30	Airborne Video System	0	0	0	0	0	0	0	0	0	0	0	0	0
31	Data Transfer Unit	0	0	0	0	0	0	0	0	0	0	0	0	0
32	Radar Altimeter System	0	0	0	0	0	0	0	0	0	0	0	4	0
33	Targeting Pod (GTP) System	0	0	0	0	0	0	0	0	0	0	0	0	0
34	Navigation Pod (VP) System	0	0	0	0	0	0	0	0	0	0	0	0	0
35	Harm Targeting System (HTS)	0	0	0	0	0	0	0	0	0	0	0	0	0
36	Weapons Delivery System	0	0	0	0	0	0	0	0	0	0	0	0	0
37	Gun System	96	0	0	0	0	0	0	0	0	0	0	0	0
38	Electronic Counter Measures	0	0	0	0	0	0	0	0	0	4	0	0	2
39	Radar Warning Receiver	0	0	0	0	0	0	0	0	0	0	0	0	0
40	Chaff/Flare Disp System	0	0	0	0	0	0	0	0	0	0	0	0	0
41	Emergency Equipment	0	0	0	2	0	0	0	0	0	0	0	0	0
42	Smoke Generation System	0	0	0	0	0	0	0	0	96	0	0	0	0
43	Explosive Devices and Components	0	0	0	0	0	0	0	0	0	0	0	0	0
44														
45		98	0	96	2	4	2	96	100	96	100	96	106	2
46														
47		0.251223	0.621521	0.136998	0.681565	0.715765	0.118215	0.168189	0.750598	0.604969	0.325374	0.785667	0.244143	0.394591
48		0.351653	0.484562	0.371892	0.159746	0.160423	0.91205	0.031981	0.047086	0.9835	0.115374	0.38401	0.780978	0.925624

missions / a_ij / rho_ij / assign / dispatch

Ready

Figure A.4 rho_ij Worksheet

Once all of the system input parameters have been entered, return to the missions worksheet by clicking on the "missions" tab at the bottom of the screen. This will return the user to the screen shown in Figure A.2. Click the button labeled "Heuristic" in the upper left-hand corner of the screen to run both the heuristic and dispatching rule. The assignment made by the heuristic will be written to the "assign" worksheet as show in Figure A.5. This format is the exact input format required by AMPL to use CPLEX to solve for the schedule. This text may be pasted into a text file to be read in by AMPL. AMPL code is also contained in this appendix.

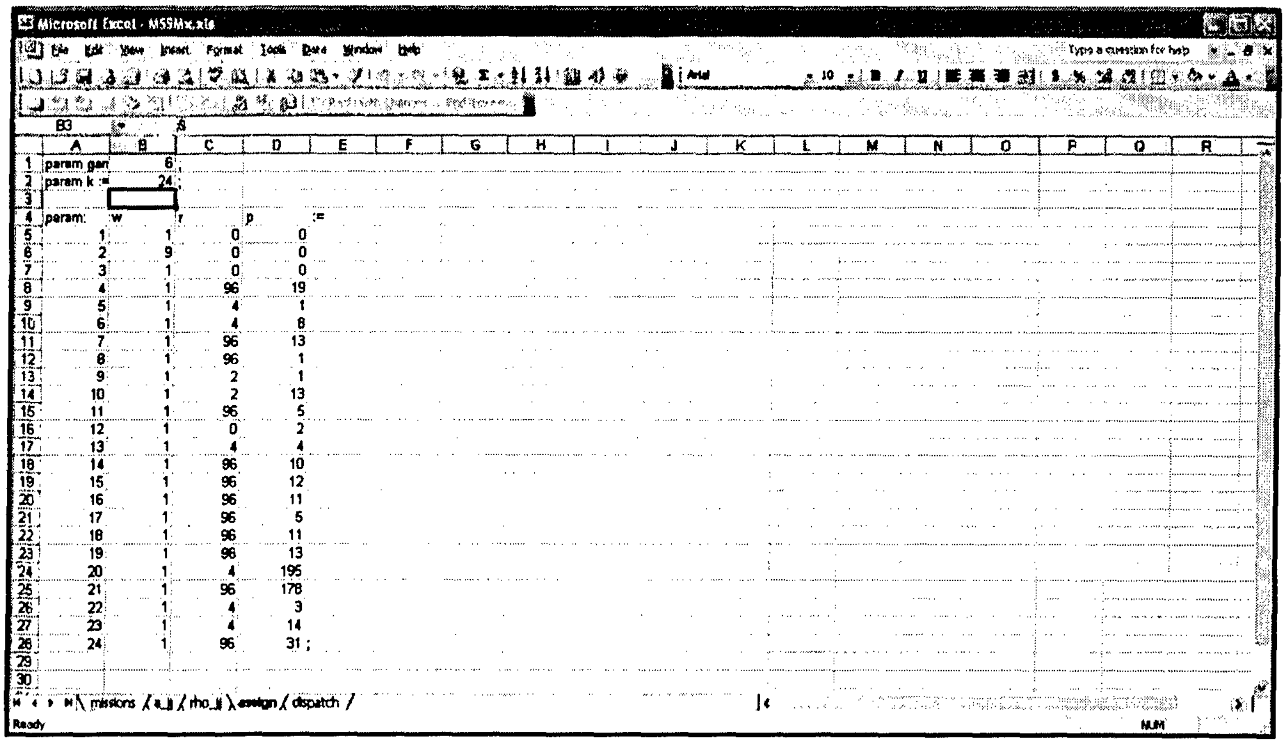
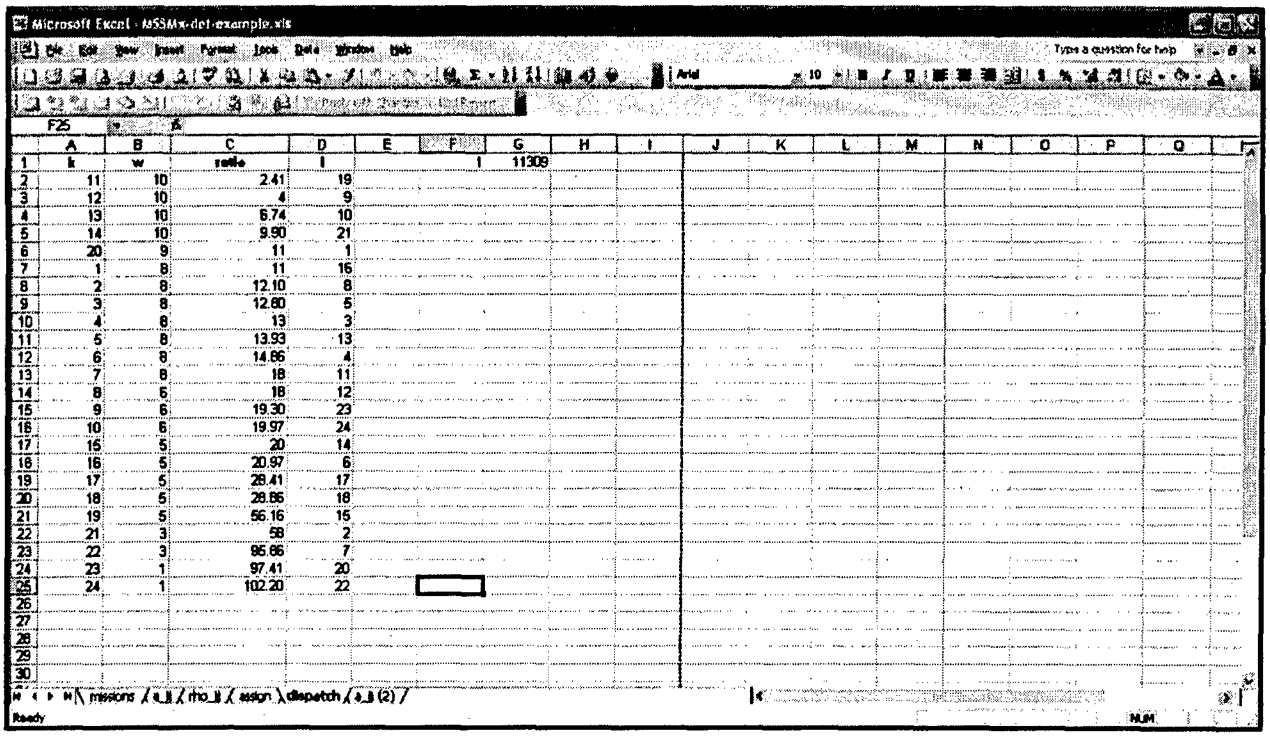


Figure A.5 Assign Worksheet

The results of the dispatching rule are written to "dispatch" worksheet show in Figure A.6. The dispatch worksheet shows the in descending order, going down the rows, the order in which the mission/system assignments are dispatched. The "k" column refers to the mission number and the "i" column refers to the system number. Mission and system numbers on the same row are assigned to each other. The "w" column displays the weight of the mission and the ratio column shows the $\sum_{j=1}^{m} a_{ij} + \sqrt{\sum_{j=1}^{m} r_{ij}}$ calculation. The objective function of the dispatching rule is displayed on the upper right-hand side with a "1" next to it indicating the first solution in the experiment.



	A	B	C	D	E	F	G
1	k	w	ratio	i		1	11309
2	11	10	2.41	19			
3	12	10	4	9			
4	13	10	6.74	10			
5	14	10	9.90	21			
6	20	9	11	1			
7	1	8	11	16			
8	2	8	12.10	8			
9	3	8	12.60	5			
10	4	8	13	3			
11	5	8	13.93	13			
12	6	8	14.66	4			
13	7	8	18	11			
14	8	6	18	12			
15	9	6	19.30	23			
16	10	6	19.97	24			
17	15	5	20	14			
18	16	5	20.97	6			
19	17	5	28.41	17			
20	18	5	28.66	18			
21	19	5	56.16	15			
22	21	3	58	2			
23	22	3	95.66	7			
24	23	1	97.41	20			
25	24	1	102.20	22			

Figure A.6 Dispatch Worksheet

AMPL Scheduling Formulation

```
param gamma;
param k;

set crew = 1 .. gamma;
set mission = 1 .. k;

param w {mission};
param p {mission};
param r {mission};

param cmax = sum {j in mission} (r[j] + p[j]);

set time = 0 .. cmax-1;

var s {mission} >=0 integer;
var x {crew, mission, time} binary;

minimize TWCT: sum {j in mission} w[j] * sum{i in crew, t in time} (t + p[j]) * x[i,j,t];

subject to one {j in mission}: sum {i in crew, t in time} x[i,j,t] = 1;

subject to two {i in crew, t in time}: sum {j in mission, tt in max(t-p[j],0) .. t-1} x[i,j,tt] <= 1;

subject to three {j in mission}: sum{i in crew, t in time} t * x[i,j,t]= s[j];

subject to four {j in mission}: s[j] >= r[j];
```